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Analysis and Implementation of a Single-Phase Line using the Bewley's Lattice Diagram

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Abstract: This paper presents the behavior of traveling waves in a single-phase transmission line. To this end, the reflected and transmitted voltages at the connection terminals between the source and the line, and between the line and the load, that is, points of discontinuity, are analyzed. The reflected and refracted voltages and currents at junctions and terminations of the lines and cables are calculated by the use of reflection and refraction coefficients. As the travelling time of the transient wave will increase, more number of reflected and transmitted components will be formed. Validation of the results and calculations performed was carried out using the ATP/EMTP software, considering the same system under analysis.
Keywords: Traveling waves, transmission line, reflection, refraction.

1. INTRODUCTION

It is well-known that high frequency transient signals will be generated whenever disturbances occur, where the power network will lose its steady state condition, resulting in a large number of cases with load drops [1].According to Travelling Waves (TW) theory, any disturbance or a sudden change in an overhead transmission line or underground cable will generate both forward and backward TWs signals propagating away from the disturbance point towards both busbars [2].

The theory of traveling waves (TWs) has gained importance year after year in power transmission grids applications, such as in TW-based protection and fault location algorithms [3]. The principle of TWs propagation in transmission lines has been known for a long time [4], but only in recent years, with the advancement of analog-to-digital converter technology, digital disturbance recorders (RDPs) and protection relays have become available of sufficient sampling rates for the application of algorithms based on TW theory [5].

The behavior of the traveling waves is strictly depending on the reflection and refraction coefficients in the sending and receiving nodes, in turn these coefficients are depending on the transmission network's parameters, such as the characteristic impedance and the impedance of the sending and receiving ends [6].

Traveling waves are electromagnetic impulses of high speed, current frequency and voltage, which originate when adisturbance occurs. For example, when a failure occurs in a transmission line, these waves will propagate towards both sending and receiving ends, until they reach a discontinuity or equipment in a substation, such as a transformer or another line with different characteristics, in the which are divided into transmitted wave and reflected wave. This phenomenon is represented by Bewley's Lattice diagrams, considered the most used tool [7].

Bewley's Lattice diagram is a graphical method that has been widely used for determining value of a TW in transient analysis [2]. Bewley's Lattice diagram is a pictorial method devised by Bewley, which shows at a glance the position and direction of motion of every incident, reflected, and transmitted wave on the system at every instant of time [1]. Further, these signals will be reflected and refracted at the points of discontinuity, i.e., fault point and busbars, until they are attenuated to a negligible value.

The salient feature of the tool is that it keeps the track for successive reflections and transmissions at various junctions for each point on them which otherwise is a quite difficult job in itself [8].

Bewley's Lattice diagram has the following properties [1]:

a) all waves travel downhill, because time always increases.
b) the position of any wave at any time can be deduced directly from the diagram.
c) the total potential at any point, at any instant of time is the superposition of all waves which have arrived at that point up until that instant of time, displaced in position from each other by intervals equal to the difference in their time of arrival.
d) The history of the wave is easily traced. It is possible to find where it came from and just what other waves went into its composition.
e) attenuation is included, so that the wave arriving at the far end of a line corresponds to the value entering multiplied by the attenuation factor of the line.

Applicability of this method has been well explored in areas like study of electromagnetic transients [9], fault location in distributed systems [10]. The detailed literature survey was carried out in [1].

The importance of analyzing traveling waves generated by disturbances such as faults or lightning in transmission lines is that they can be used as criteria for their detection, location and subsequent release [11].

Fault location using fault transients and based on traveling waves theory has been successfully applied as a unit or double ended scheme on extra high-voltage (EHV) transmission lines. A single end fault location scheme is also possible when the current and voltage transients are available at the relaying point. A single ended fault location scheme has its origin in offline traveling-wave fault locators, which, inject a signal and fault locate from the time it takes the signal to reflect back from the fault location [1].

The basic principle of this method can be well explained using Bewley's Lattice diagram as illustrated in Fig. 1 [6].

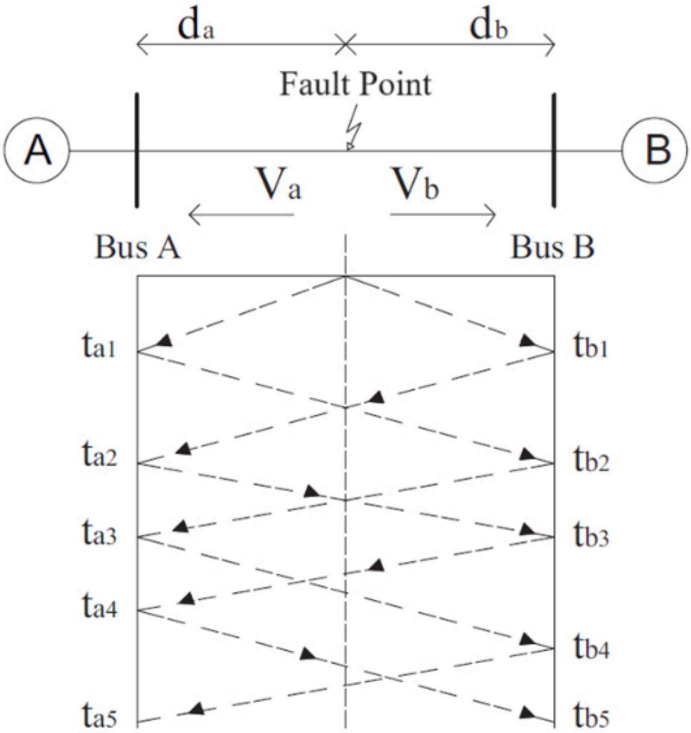


Figure 1. Representation of the Bewley's Lattice diagram when a fault occurs in the middle of a transmission line [6]

The Bewley's Lattice diagram developed by L. V. Bewley [7] organizes the reflections that occur during transients in the transmission line. For the Bewley's Lattice diagram, the vertical scale represents time t in units of s and the diagonal lines represent traveling waves. Each reflection is determined by multiplying the incident wave arriving at one end by the reflection coefficient at that end, as illustrated in Fig. 1.

The voltage $u(x,t)$ at any point x and t on the diagram is determined by adding all the terms directly at that point [12].

To exemplify what was said previously [6], Fig. 1 supposes a disturbance at a distance x in a transmission line, where it can be seen that the signals seen from terminal A allow obtaining information about the phenomenon of traveling waves. Furthermore, it is possible to observe in Fig. 1, when the disturbance occurs at the intermediate point of the line, a voltage wave V_a propagates towards terminal A, covering a distance in a time t_{a1} and another voltage wave V_b propagates in towards terminal B, traveling a distance d_b in time t_{b1}, when it reaches terminal B it is reflected and returns to the point where the disturbance is located. At this point, this voltage wave is reflected and returns at time t_{b2} and another wave is refracted and heads towards terminal A at time t_{a2} and so on.

The waves reaches both the busbars after a certain time delay which is actually equal to the time required by the wave to travel the distance with speed of 3 x 10^8 m/s i.e. speed of light. Upon reaching the busbar, some part of the wave is transmitted to the other medium and rest is reflected back to the same medium. The value is decided by the impedance value of both mediums present on both side of concerned busbar. The same pattern gets repeated when this new generated waves represented subsequent arrows reaches again to the busbars [1].

These travelling waves are generated by the fault contain abundant fault information such as fault direction, fault location, time of fault occurring and so on [13][14] and they can be used as the fault detection criteria.

2. TRANSITIONAL REGIME IN TRANSMISSION LINES

2.1 Propagation of traveling waves in transmission lines

Understanding the propagation of TWs in transmission systems [3] begins with modeling transmission lines using distributed parameters. In fact, it is from this modeling that it becomes possible to simulate and understand the phenomena transients arising from the propagation of TWs in transmission lines [15].

In Fig. 2, the incremental equivalent circuit is presented of a LT, where Δx represents the length of the circuit incremental, R the resistance, L the inductance, G the conductance and C the capacitance of the incremental circuit, all representing the concept of parameters per unit length [3].

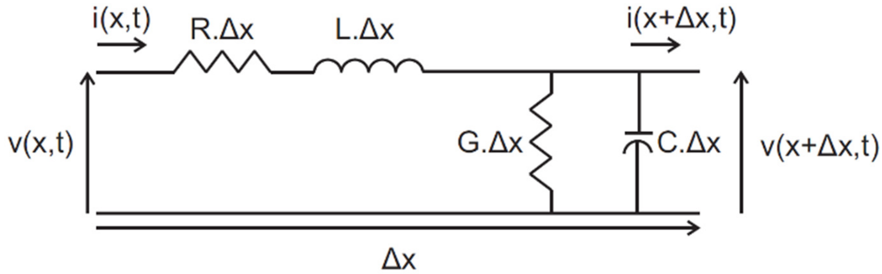


Figure 2. Equivalent incremental circuit of an transmission line segment [3]

Assuming sinusoidal steady state, the transmission line equations associated with a single conductor can be written in the phasor domain (or frequency) such as:

$$\frac{\partial V(x,t)}{\partial x} = -(R + j\omega L) I(x,t) = -Z(\omega) I(x,t) \tag{1}$$

$$\frac{\partial I(x,t)}{\partial x} = -(G + j\omega C) V(x,t) = -Y(\omega) V(x,t) \tag{2}$$

Deriving the previous equations with respect to x, is obtain the following pair of equations:

$$\frac{\partial^2 v(x,t)}{\partial x^2} = -Z(\omega) Y(\omega) V(x,t) \tag{3}$$

$$\frac{\partial^2 I(x,t)}{\partial x^2} = -Y(\omega) Z(\omega) I(x,t) \tag{4}$$

For a single-phase transmission line $Z(\omega)Y(\omega) = Y(\omega)Z(\omega)$. Soon:

$$\frac{\partial^2 v(x,t)}{\partial x^2} = \gamma^2 V(x,t) \tag{5}$$

$$\frac{\partial^2 I(x,t)}{\partial x^2} = \gamma^2 I(x,t) \tag{6}$$

Where

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \tag{7}$$

The variable γ is known as constant or programming coefficient. This coefficient can also be expressed as follows:

$$\gamma = \sqrt{Z \cdot Y} = \alpha + j\beta \tag{8}$$

The solution to equations (5) and (6) has the form:

$$V(x,t) = V_1 e^{-\gamma x} + V_2 e^{\gamma x} \tag{9}$$

$$I(x,t) = I_1 e^{-\gamma x} + I_2 e^{\gamma x} \tag{10}$$

The equations (9) and (10) represent the propagation of regressive and progressive TWs in LTs after a sudden voltage variation in step form. This phenomenon is the same as that observed during faults or switching maneuvers, which launch TWs that propagate in the system with a speed defined based on the LT parameters. Based on this concept, it is noted that TWs have a simultaneous variation in space and time, a fact that motivates the use of Lattice diagrams in study of the propagation of TWs in LTs. In fact, this type of diagram is two-dimensional, so it makes use of two axes orthogonals that represent time and space. This way, it becomes possible to evaluate in detail the spread of TWs in LTs, facilitating the understanding of transients typically observed in transmission systems [3].

Note that it is possible to determine a relationship between the constants V_1 and I_1, V_2 and I_2. Replacing (10) in (1):

$$\frac{\partial V(x,t)}{\partial x} \cdot \frac{1}{Z} = -\left(I_1 e^{-\gamma x} + I_2 e^{\gamma x}\right) = -I(x,t) \tag{11}$$

Deriving equation (9) as a function of x:

$$\frac{\partial V(x,t)}{\partial x} = -\gamma V_1 e^{-\gamma x} + \gamma V_2 e^{\gamma x} \tag{12}$$

Replacing (12) into (11):

$$I(x,t) = \frac{\gamma}{Z}\left(V_1 e^{-\gamma x} + \gamma V_2 e^{\gamma x}\right) \tag{13}$$

However, the characteristic admission is given by:

$$\frac{\gamma}{Z} = \frac{\sqrt{Z \cdot Y}}{Z} = \frac{\sqrt{Z \cdot Y}}{\sqrt{Z^2}} = \sqrt{\frac{Z \cdot Y}{Z^2}} = \sqrt{\frac{Y}{Z}} = Y_C \tag{14}$$

Therefore, the characteristic impedance will be:

$$Z_C = \frac{1}{Y_C} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \left|Z_C\right| e^{j\theta} \tag{15}$$

2.2 Reflection and transmission of traveling waves

Both the polarity and the amplitude of a wave measured at a given terminal are related to the reflection and transmission coefficients at measurement points of the monitored system, which are represented in this work by the variables Γ and T, respectively [3].

To understand the behavior of traveling waves in transmission lines, considering the application of the Laplace Transform, it is necessary to carry out the study of electrical circuits for a transmission line and apply Kirchhoff's laws. In Fig. 3, a lossless transmission line is illustrated, which consists of one phase and two wires, of length l and a load impedance $Z_R(s)$. In addition, a Thévenin impedance $Z_S(s)$ is observed in the sending node and a characteristic impedance Z_C associated with the line. This lossless line assumption is widely used in power systems for studies of transients in transmission lines. In addition, the conditions for the propagation study are summarized, when a short circuit and an open circuit occur in the parameter transmission line distributed [6].

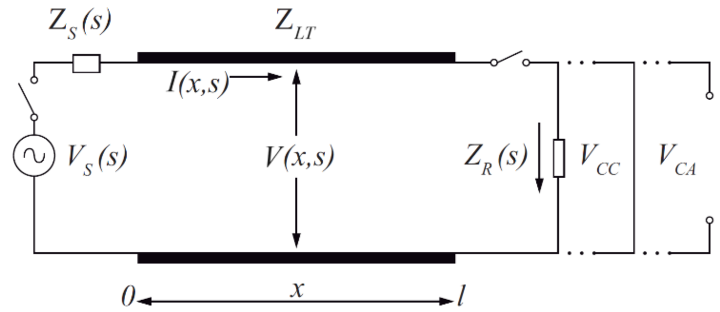


Figure 3. Transmission line in the Laplace domain [6]

According to [6] the equations that describe the behavior of the lossless transmission line with distributed parameters are:

$$V_x(x,t) = -L \cdot I_t(x,t) \tag{16}$$

$$I_x(x,t) = -C \cdot V_t(x,t) \tag{17}$$

Where $u_x(x, t)$ and $i_x(x, t)$ are the partial derivatives with respect to x of the voltage and current respectively, $u_t(x, t)$, and $i_t(x, t)$ are the partial derivatives with respect to t of the voltage. on and current respectively and L and C are the inductance and capacitance of the transmission line. Equations (16) and (17) are reduced to the following voltage traveling wave equation [6].

$$V_{tt}(x,t) - v^2 V_{xx}(x,t) = 0 \tag{18}$$

As indicated by, considering the application of the Laplace Transform, it is assumed that for the solution of equation (18) the initial conditions are equal to zero, so the general solution is as [6]:

$$V(x,s) = A(s)e^{-\frac{s}{v}x} + B(s)e^{\frac{s}{v}x} \tag{19}$$

However, when the initial conditions are different from zero, according to the general solution of (18) in the Laplace domain has the form [6]:

$$V(x,s) = A(s)e^{-\frac{s}{v}x} + B(s)e^{\frac{s}{v}x} - \frac{1}{sV}\int_0^x \left[sV(y,0) + V_t(y,0)\right]\left(\frac{e^{-\frac{s}{v}x} - e^{-\frac{s}{v}x}}{2}\right)dy \tag{20}$$

The result is obtained [6]:

$$V(x,s) = -\frac{V_S(s)\left[1-\Gamma_S(s)\right]}{2\left(1-\Gamma_S(s)\Gamma_R(s)e^{\frac{2s\ell}{v}}\right)}\left(e^{-\frac{s}{v}x} + \Gamma_R(s)e^{-\frac{2s\ell}{v}}e^{\frac{s}{v}x}\right) \tag{21}$$

$\Gamma_S(s)$ and $\Gamma_R(s)$ are the reflection coefficients at the sending and receiving nodes, respectively [6]. The Table 1 presents the reflection coefficients and transmission coefficients for voltage and current considering the system in Fig. 3.

Table 1 Expressions for reflection and transmission coefficients

	Reflection Coefficients	Transmission Coefficients
Voltage	$\Gamma_{Sv}=\frac{Z_S-Z_{LT}}{Z_S+Z_{LT}}$ $\Gamma_{Rv}=\frac{Z_R-Z_{LT}}{Z_R+Z_{LT}}$	$T_{Sv}=\frac{2Z_S}{Z_S+Z_{LT}}$ $T_{Rv}=\frac{2Z_R}{Z_R+Z_{LT}}$
Current	$\Gamma_{Si}=\frac{Z_{LT}-Z_S}{Z_S+Z_{LT}}$ $\Gamma_{Ri}=\frac{Z_{LT}-Z_R}{Z_R+Z_{LT}}$	$T_{Si}=\frac{2Z_{LT}}{Z_S+Z_{LT}}$ $T_{Ri}=\frac{2Z_{LT}}{Z_R+Z_{LT}}$

3. APLICATION OF BEWLEY'S LATTICE DIAGRAM IN A SINGLE-PHASE TRANSMISSION LINE

The transmission line analyzed is single-phase and its conductors are positioned at a height of 10 meters, the radius of the conductors is 0.005 meters and the impedance is equal to 497.6 ohms. Other information is presented in Fig 4.

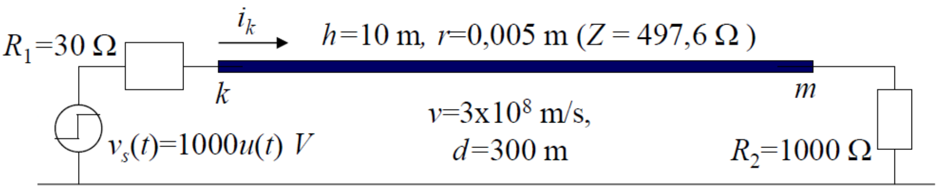


Figure 4. Single-phase transmission line

The value of the incident wave (v_{ICD}) and transit time (τ) are 943.13V and 1μs, respectively. Considering the electrical system under study, voltages at nodes k and m were obtained from the reflection and transmission coefficients indicated in Table 2.

Table 2 Expressions for reflection and transmission coefficients

Time	Node k	Node m
0τ	v_{ICD} = v_{ICDk1} = 943.13 V	
1τ		v_{Tm1} = 1259.6 V $v_{\Gamma m1}$ = 316.42 V
2τ	v_{Tk2} = 35.98 V $v_{\Gamma k2}$ = - 280.44 V	
3τ		v_{Tm2} = - 374.53 V $v_{\Gamma m2}$ = - 94.09 V
4τ	v_{Tk3} = - 10.70 V $v_{\Gamma k3}$ = 83.39 V	
5τ		v_{Tm3} = 111.37 V $v_{\Gamma m3}$ = 27.98 V
6τ	v_{Tk4} = 3.18 V $v_{\Gamma k4}$ = - 24.80 V	
7τ		v_{Tm4} = - 33.12 V $v_{\Gamma m4}$ = - 8.32 V
8τ	v_{Tk5} = - 0.95 V $v_{\Gamma k5}$ = 7.37 V	
9τ		v_{Tm5} = 9.84 V $v_{\Gamma m5}$ = 2.47 V
10τ	v_{Tk6} = 0.28 V $v_{\Gamma k6}$ = - 2.19 V	
11τ		v_{Tm6} = - 2.92 V $v_{\Gamma m6}$ = - 0.73 V

In Fig. 5, the Bewley's Lattice diagram is represented.

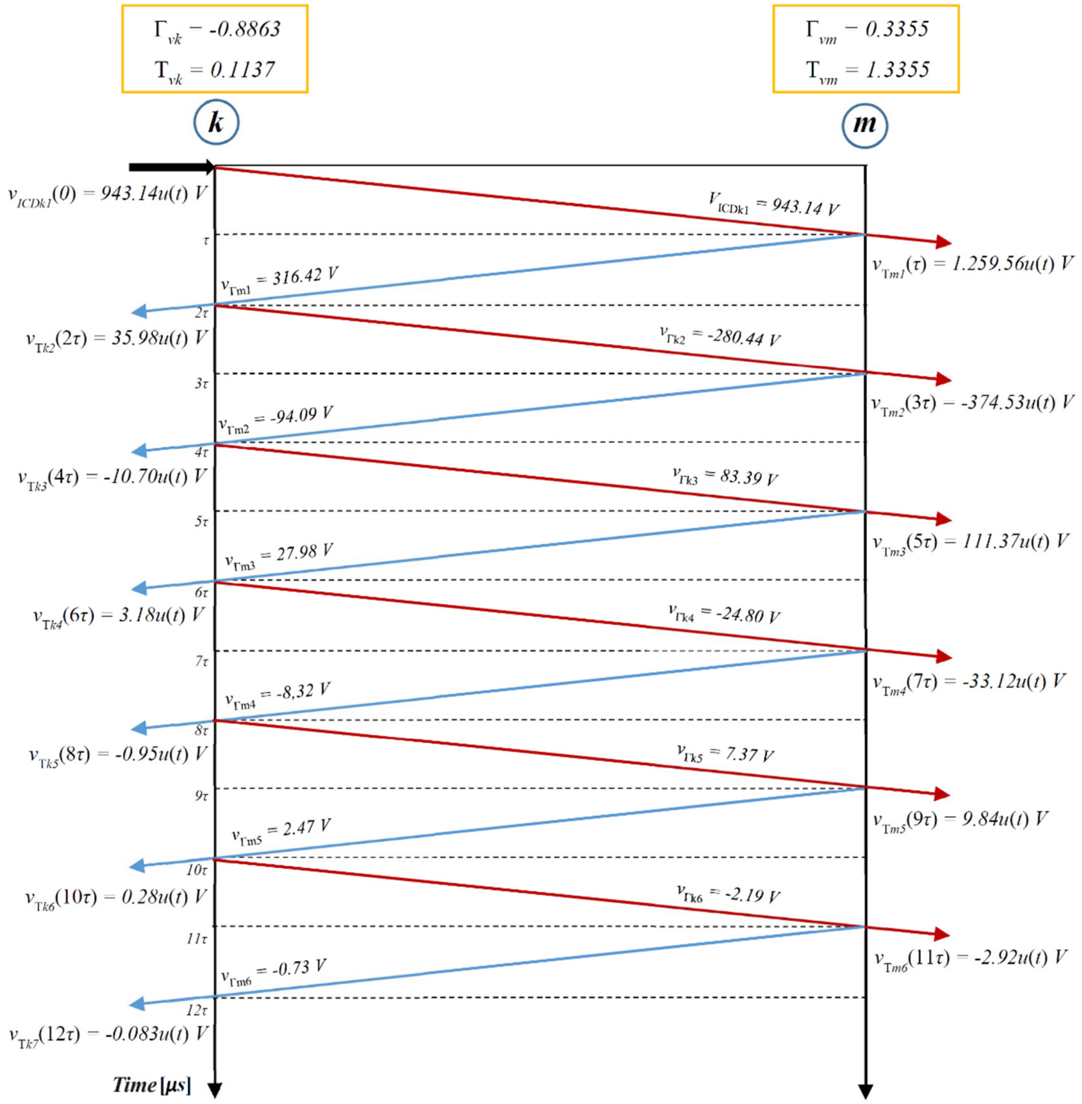
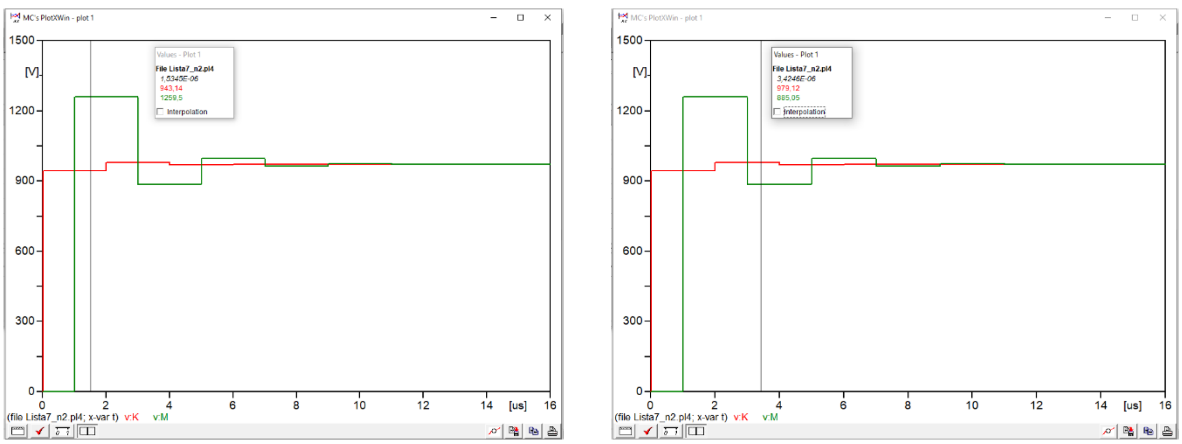


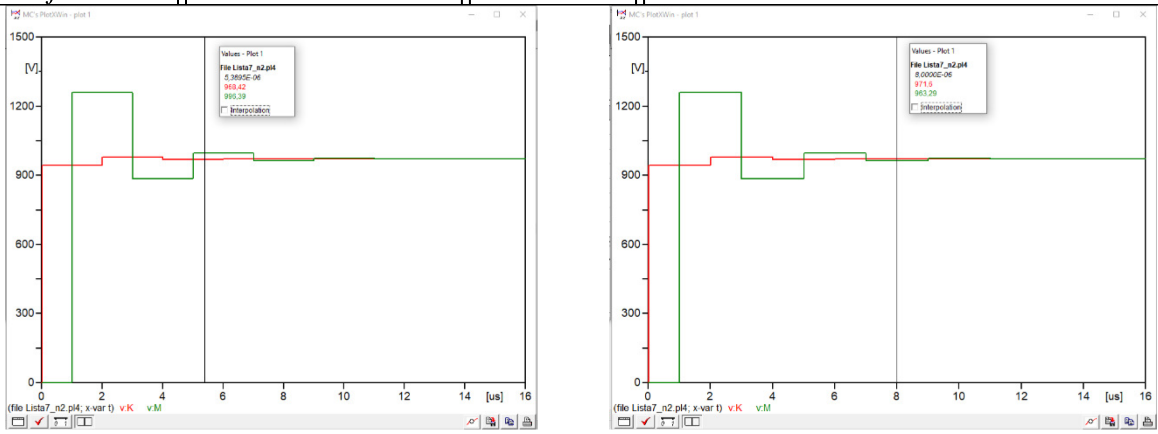
Figure 5. Bewley's Lattice diagram showing reflections and transmissions in single-phase transmission line

Fig. 6, Fig.7 and Fig. 8 were obtained considering the same electrical system implemented in ATP/EMTP, thus validating the values obtained through calculations and equations.



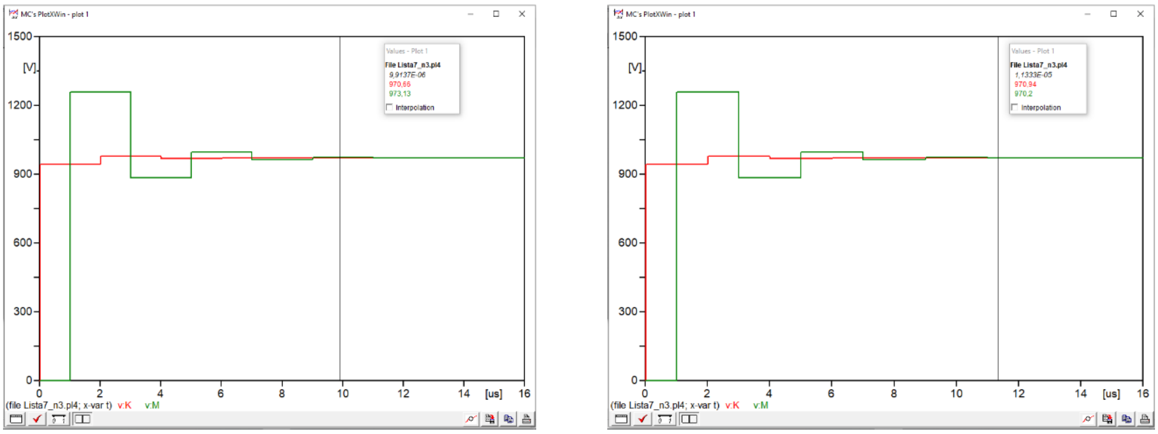
(a) (b)

Figure 6. Voltages waves (v_Γ and v_T): (a) $t = 0\tau$; (b) $t = 3\tau$



(a) (b)

Figure 7. Voltages waves (v_Γ and v_T): (a) $t = 4\tau$; (b) $t = 7\tau$



(a) (b)

Figure 8. Voltages waves (v_Γ and v_T): (a) $t = 8\tau$; (b) $t = 11\tau$

4. CONCLUSION

This work evaluated the application of the Bewley's Lattice diagram in a single-phase transmission line. It is important to highlight that to use the diagram, the reflection and transmission coefficients of a line of distributed parameters were mathematically determined, considering a Laplacian solution of the universal line model.

To evaluate the behavior of traveling waves in a single-phase electrical system, the reflected and transmitted voltages in this system were calculated, and graphic representations of these waves were made using the Bewley's Lattice diagram. To validate the calculations and results, general equations were obtained that describe the behavior of traveling waves in the system under study. Later, system simulations were carried out in the ATP/EMTP software and the same results were found, in addition to the representation of the voltage wave. It is concluded that the work presented satisfactory and adequate results for the study carried out.

Furthermore, it is clear, through the literature review, that the Bewley's Lattice diagram is being implemented in the protection of transmission lines that use algorithms based on traveling waves.

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