International Journal of Latest Engineering and Management Research (IJLEMR)
ISSN: 2455-4847
www.ijlemr.com || Volume 04 - Issue 12 || December 2019 || PP. 14-19

Fractional PI Controller for a Heat Exchanger system

[1]Shanti Sankara Krishnan, [2]Sandhya.kumar, [3]Imran Sayeed, [4]Apurva Sawant
[1,4]*Faculty, EXTC department*
[2,3]*Asst. Prof., EXTC department*
Vidyalankar Polytechnic, WadalaVidyalankar Polytechnic, Wadala

Abstract: Fractional-order (FO) model provides a more realistic representation of the real world systems. [1]. Recently, it has been shown that FO controllers provide a more efficient, guaranteed, robust control to both FO and IO systems. FOPID controller parameters are composed of the proportionality constant, integral constant, derivative constant, derivative order and integral order, and its design is more complex than that of conventional integer order proportional integral derivative (PID) controller. In this paper, a FOPI controller design method that achieves user specified gain and phase margins which is easy and simple to implement is proposed for a Heat exchanger system involving delay.

General Terms: Stability, Closed loop response, PID parameters
Keywords: Fractional-order controllers, PID controllers, Gain margin,phase margin, Time delay systems

1. Introduction

Heat exchangers are important thermal systems with widespread applications, including: air conditioning, power generation, manufacturing, etc. The main goal of these devices is to maintain precise temperature conditions by controlling outlet temperatures of the working fluids in response to the operating conditions in a particular application. It is, thus, crucial to predict and control the behavior of these thermal devices. This calculation is difficult from a first-principles standpoint; geometry, turbulence, temperature dependent properties all add to the complexity of the problem. As a result, e.g., predictions are typically based on mathematical models derived from experimental data for specific heat exchangers. For steady state conditions, correlation equations in terms of Nusselt and Reynolds numbers are common [2]. However, modeling the dynamic behavior of a heat exchanger typically requires solving either partial differential equations (PDEs) or systems of ordinary differential equations (ODEs), which are computationally expensive and not suitable for real-time control purposes. Therefore, there is a need for compact and accurate mathematical models of these systems.

A shell and tube heat exchanger consist of a bundle of tubes enclosed within a cylindrical shell. One fluid flow through the tubes and a second fluid flows within the space between the tubes and the shell. Heat is thus transferred from one fluid to the other through the tube walls, either from tube side to shell side or vice versa. They can further be classified according to their flow arrangement. Most shell and tube heat exchangers are 1,2 or 4 pass designs on the tube side depending upon the number of times the fluid in the tubes passes through the fluid in the shell. Counter-flow and parallel-flow are the two primary flow arrangements in heat exchanger. In Counter current mode, the hot fluid enters from one end of the exchanger and the cold fluid from the opposite end. In Co-current (Parallel) mode the flow of the hot and the cold fluid are taking place in the same direction. The outlet temperature of the shell and tube heat exchanger system has to be kept at a desired set point according to 18 B. Girirajan and D. Rathikarani., 2017/Advances in Natural and Applied Sciences. 11(9) July2017, Pages: 17-25 the process requirement. Due to nonlinear nature, shell and tube heat exchanger system is hard to model and control using conventional methods. The integer order PID controller completely deals with the system dynamics whose behaviors are described by integer order differential equations. The closed system with this controller exhibits poor settling time due to its integer values of control parameters for a system involving non-integer values. Moreover, it has insufficient control parameters for a system such as heat exchanger involving time series of heat transfer. The real physical systems are well characterized by fractional order differential equations involving non integer order derivatives. This gives the option of fractional order dynamic systems and controllers based on fractional order calculus.[4]

The integer order PID controller completely deals with the system dynamics whose behaviors are described by integer order differential equations. The closed system with this controller exhibits poor settling time due to its integer values of control parameters for a system involving non-integer values. Moreover, it has insufficient control parameters for a system such as heat exchanger involving time series of heat transfer. The real physical systems are well characterized by fractional order differential equations involving non integer order derivatives. This gives the option of fractional order dynamic systems and controllers based on fractional order calculus.[4]

A simple FOPI controller design method that achieves user-specified gain and phase margins was proposed for a linear system involving delay [3]. Unlike other existing methods in the same category where assumptions on the process dynamics are made to simplify the nonlinear problem encountered in computation, this method could yield exact solution for a general linear process. This also led to a simple solution to the gain and phase margin problem. Gain and phase margins are typical control loop specifications associated with the frequency response technique. Not only do they serve as important indicators of system, they also reflect robustness on the performance and stability of the system and thus are widely used for controller designs.

2. Fractional Calculus

2.1 A brief Introduction

Although the fractional order calculus is a 300-years old topic, the theory of fractional order derivative was developed mainly in the 19th century. Fractional calculus is a generalization of integration and differentiation to a fractional, or non-integer order fundamental operator aD_t^{α} where a and t are the lower/upper bounds of integration and α the order of the operation.

There are three main advantages for introducing fractional order calculus to control engineering:

(1) adequate modeling of control plants dynamic features.

(2) effective and clear-cut robust control design.

(3) reasonable realization by approximation [9]. The fractional calculus (FC) was unexplored in engineering, because of its inherent complexity, the apparent self-sufficiency of the integer order calculus (IC), and the fact that it does not have a fully acceptable geometrical or physical interpretation. The developmentof the fractional calculus was mainly in the hands of mathematicians. This led to a number of competing definitions of the derivative and integral operators. Its applications in engineering was thus delayed because of these multiple definitions. In the latter years FOC however attracted engineer's attention, because it can describe the behavior of real dynamical systems in compact expressions. Many natural phenomena may be better described by a FOC formulation, because it takes into account the past behavior and it is compact when expressing high-order dynamics. Some common definitions of FOC are:

[1] ***Grunwald-Letnikov:***

$$D^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{m=0}^{t-\frac{a}{h}} \frac{\Gamma(\alpha+m)}{m!\Gamma(\alpha)} f(t-mh) \ldots (2.1)$$

The Grunwald-Letniko definition is a generalization of the common derivative.

[2] ***Riemann-Liouville***

$$_{RI}D^{\alpha}\, f(t) = D^{\alpha} J^{m-\alpha} = \frac{d^m}{d^{tm}} 1/\Gamma(m-\alpha) \int_{\mathbf{0}}^{\mathbf{t}} \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \ldots (2.2)$$

where (m-1)<α<m, $m \in n$

[3] Caputo

$$_{c}D^{\alpha} f(t) = J^{m-\alpha} D^{m} = \frac{1}{T(m-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \ldots (2.3)$$

where (m-1)<α<m, $m \in n$

The global definition has a convolutional format. It is worth mentioning here that from the pure mathematical point of view there are several ways to interpolate between integerorder multiple integrals and derivatives. The most widely known and precisely studied is the Riemann Liouville definition of fractional derivatives. The main advantage of Caputo's definition in comparison with the Riemann Liouville definition is that it allows consideration of easily interpreted conventional initial conditions such as y(0) = y0, y′ (0) = y (1) etc. Moreover, Caputo's derivative of a constant is bounded (namely, equal to zero), while the Riemann Liouville derivative of a constant is unbounded at t =0.

2.2 Fractional order systems

2.2.1 Introduction

A fractional order differential equation, provided both the signals u(t) and y(t) are relaxed at t = 0, can be expressed in a transfer function form.

$$P(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta m} + b_{m-1} s^{\beta^{m-1}} + \cdots \ldots + b_0 s^{\beta^0}}{a_n s^{\alpha n} + a_{n-1} s^{\alpha n-1} + \cdots \ldots + a_0 s^{\alpha^0}} \ldots (2.4)$$

where a_k(k = 0, ...n), b_k α(k = 0, ...m) are constant, and α_k (k = 0, . . . n), β_k(k = 0, ...m) are arbitrary real or rational numbers and without loss of generality they can be arranged as $\alpha_n > \alpha_{n-1} \ldots > \alpha\ 0$ and $\beta n > \beta_{n-1} \ldots > \beta_0$.

We propose a generalization of the PID controller, which can be called the $PI^{\alpha}D^{\beta}$-controller because of involving an integrator of order α and a differentiator of order β.

$u(t) = K_p e(t) + K_I J^{\alpha} e(t) + K_D D^{\beta} e(t) \ldots (2.5)$

The equation for the $PI^{\alpha}D^{\beta}$ controller output in the time domain is

The transfer function of such a controller has the form given by

$$G(s) = \frac{U(s)}{E(s)} = K_p + K_I s^{-\alpha} + K_D s^{\beta} \ \ldots (2.6)$$

G(s) is the transfer function of the controller. e(t) is an error and u(t) is controller's output. Taking α = 1 and β = 1, we obtain a classic PID-controller. α = 1 and β = 0 give a PI-controller. α = 0 and β = 1 give a PD-controller. α = 0 and β = 0 give a gain. All these classical types of PID-controllers are the particular cases of the fractional $PI^{\alpha}D^{\beta}$ controller [4]. However, the $PI^{\alpha}D^{\beta}$ controller is more flexible and gives an opportunity to better adjust the dynamical properties of a fractional-order control system.

2.2.2 Oustaloup Recursive Approximation:

FO differentiator/Integrator cannot be simulated directly. Various methods are available for IO approximation of these fractional terms. OustaloupRecursive Approximation (ORA) is one of them. Oustaloup Recursive Approximation is widely used to find a rational integer-order approximation for fractional-order integrators and differentiators of the form sα, The approximation algorithm presented by Oustaloup is widely used where a frequency band of interest is considered within which the frequency domain responses should be fit by a bank of integer order filters to the fractional order derivative . Suppose that the frequency range to be fit are given by ωa, ωb the term s/ωu can be substituted with

$$C_0 \frac{1 + s/\omega_b}{1 + s/\omega_\hbar} \ldots (2.7)$$

Where $\sqrt{\omega_b \omega_\hbar} = \omega_u \ \ldots (2.8)$

And $C_o = \frac{\omega_b}{\omega_u} = \frac{\omega_u}{\omega_\hbar} \ldots (2.9)$

Unfortunately, the fitting quality around the frequency band boundaries, and,may not be satisfactory. The Oustaloup's approximation model to a fractional order differentiator s^{α} can be written as

$$\hat{H}(s) = \left(\frac{\omega_\hbar}{\omega_b}\right)^{\alpha} \prod_{k=-N}^{N} \frac{1 + \frac{s}{\omega'_k}}{1 + \frac{s}{\omega_k}} \ldots (2.10)$$

$$\omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+1/2-\alpha/2}{2N+1}}$$

$$\omega^k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+1/2+\alpha/2}{2N+1}}$$

are respectively the zeros and poles of rank k. And 2N + 1 is the total number of zeros or poles. The quality of the Oustaloup's approximation method may not be satisfactory in high and low frequency bands near the fitting frequency bounds.

2.2.3 ALGORITHM FOR THE DESIGN OF FOPI CONTROLLER

Design equations:

Let us denote the process and controller transfer function by P and G, as shown in the .In the design of controller, we have considered aspects of frequency domain gain margin (Am) and phase margin (Φ_m), which is mainly to ensure the stability of feedback control system. In other words, the controller must meet the following relationship:

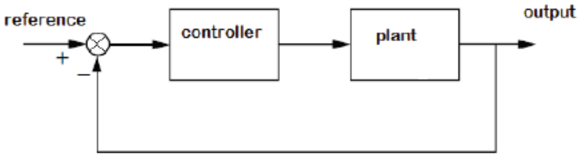

Fig. 1 Feedback Control System

$$arg\ P\ (j\omega_{pc})G(j\omega_{pc}) = -\pi \ldots (2.12)$$

$$A_m = \frac{1}{P(j\omega_{pc})G(j\omega_{pc})} \ldots (2.13)$$

$$G(j\omega_{gc})P(j\omega_{gc}) = 1 \ldots (2.14)$$

$$\phi_m = arg[\,G(j\omega_{gc})P(j\omega_{gc})] + \pi \ldots (2.15)$$

From the analysis of stability of feedback control system, we have found the following new equations. Refer paper [3] for the design values of α, Kp, and Ki.

$$\left(\frac{\Im(-\frac{1}{A_m P(j\omega_{pc})})}{\Im(-\frac{e_X^{-j\phi m}}{P(j\omega_{gc})})} \right) = a \ldots (2.16)$$

$$\frac{log\ a}{log\frac{\omega_{gc}}{\omega_{pc}}} = \alpha \ldots (2.17)$$

$$K_i = -\left(\frac{\omega_{pc}^{\alpha}}{sin(\alpha\pi/2)}\right)\Im\left(\frac{-1}{A_m P(j\omega_{pc})}\right) \ldots (2.18)$$

$$K_p = \Re\left(\frac{-1}{A_m P(j\omega_{pc})}\right) - \frac{K_p}{\omega_{pc}^{\alpha}} cos(\alpha\pi/2) \ldots (2.19)$$

The three unknown equations for *α*, *Ki*, *Kp*are given above.

2.2.4 Tuning procedure:

Given the plant *P*(*s*) or *P*(*jω*), The gain margin A_m and phase margin ϕ_m are assumed for the design. The FOPI parameters can be tuned to meet both gain margin A_m and phase margin ϕ_m in the following way:

- Obtain the process phase crossover frequency ω_{pc} from open loop TF $G(j\omega)P(j\omega)$.
- Check whetheror not the following equation issatisfied.

$$\Im(-\frac{e_X^{-j\phi_m}}{P(j\omega_{gc})}) = \Im(-\frac{1}{A_m P(j\omega_{pc})}) \ldots.(2.20)$$

If not, either reduce ω_{pc} or modify thegain/phase margin to meet (2.20)

- Search from $\omega = \omega_{pc}$ down towards $\omega = 0$ for the frequency ω_{gc} that satisfies (3.5).
- Calculate α from (2.17).
- Calculate K_i from (2.18).
- Calculate K_p from (2.19).
- Approximate FO integrator α by using ORA using (2.17).
- Simulate using Simulink environment of MATLAB.

3. Case study: Heat exchanger system:

Consider a heatexchanger system described by the transfer function [5]

$$P(s) = \frac{3e^{-10s}}{(4s+1)^2}$$

Desired specification:

1. Gain margin Am=6db
2. Phase margin □$_m$ = 60^0

The phase crossover frequency (ω_{pc}) obtained from open loop TF G(jω)(jω) is 0.1898 rad/sec. The gain crossover frequency (ω_{gc}) is obtained by satisfying the 3.26). The ω_{gc} obtained is 0.1210 rad/sec. The power α of FOPI is obtained by using (2.17). The integral gainKi is obtained by using (2.18). The proportional gain $Kp$is obtained by using (2.19). The parameters of FOPI controller are:

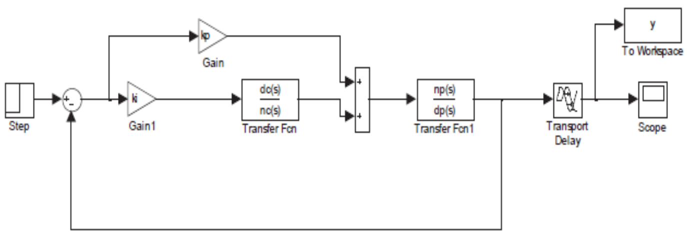

Fig. 2 Simulation of FOPI Controller

1. α = 0.7054
2. Ki = 0.0035
3. Kp = 0.1362

The fractional order term cannot be simulated directly. The integer order approximation of $s^{0.7054}$ using ORA is given by (2.18). (The frequency band is 0.01 rad/sec to 100 rad/sec). The IO approximation of s^{α} is given by assuming the following change in the controller equation as:

$$G(s) = Kp + \frac{Ki}{s^{\alpha}} = Kp + \frac{Ki}{s} s^{1-\alpha}$$

The term $s^{1-\alpha}$ is approximated. The second order IO approximation of $s^{0.7054}$i.e $s^{1-0.7054}$i.e $s^{0.2946}$ is obtained by 2.18

$$G_{approxFOPI}\ (s) = \frac{nc\,(s)}{dc\,(s)} = \frac{3.883s^2+19.9s+1}{s^2+19.9s+3.883}$$

The IO approximation of FOPI controller is given by:

$$G_{approx}\ (s) = \frac{0.5289s^3+2.714^2+0.2056s+0.01354}{3.883s^3+19.9s^2+s}$$

The designed FOPI controller is approximated by IO approximation and is simulated by using simulink environment of MATLAB. The simulink block diagram is as shown in figure (4.1). The Bode plot of open loop system with designed FOPI controller is as shown in Fig.4.3. The gain and phase margins of 6.0445 dB and 62.64240 are achieved using designed FOPI controller. The phase and gain cross over frequencies are 0.1730 rad/sec and 0.0234 rad/sec respectively. The closed loop step response is as shown in Fig.4.6. The closed loop response settles to the desired setpoint, means there is no off set. Thus we have implemented the FOPI Controller using simulink environment of MATLAB. The design equations are used and the response is given.

4. Conclusion and Future Work

A simple method for the design of FOPI controller that achieves exact gain and phase margin is proposed. It is based on the utilization of frequency response of the process.The values of α, K_i, K_p is calculated and s−α is approximated using ORA method.Simulation results have been presented to demonstrate the effectiveness of the method.From the results obtained, it is observed that the FOPI controller tuning method ensurethat the given gain and phase margins are achieved. It is also observed that FOPIcontroller achieves better control performance with the proposed design method. Fromthe response it is observed that the closed loop response does not settle to final value.

This error is because of the approximation.A comparison can be done between the integer order response & the fractional response.The work can be extended to design a FOPID controller.

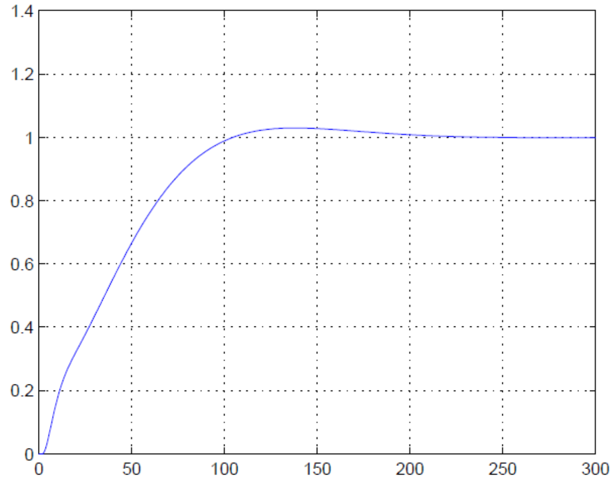

Fig. 3 Closed loop step response of the system with FOPI controller

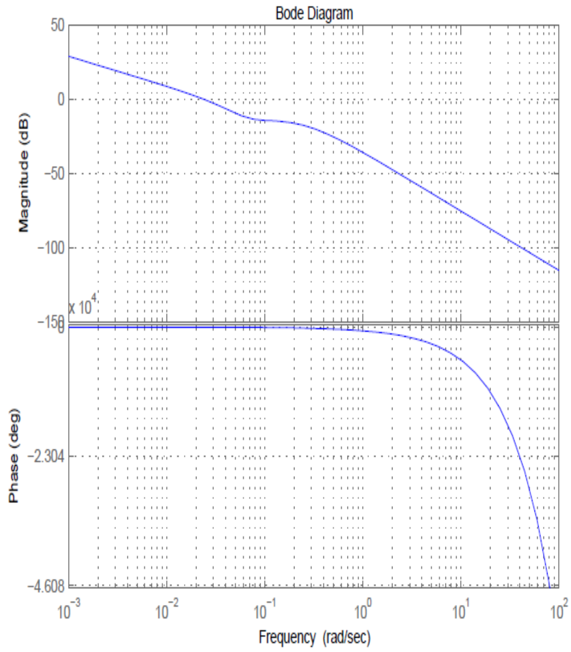

Fig.4 Bode plot of the system with designed FOPI controller

5. References

[1]. Fractional-order systems and $PI^{\lambda}D^{\beta}$ controllers. Igor Podlubny. IEEE, (1999).

[2]. Heat rate predictions in humid air-water heat exchangers using correlations and neural networks,A. Pacheco-Vega, G. Diaz, M. Sen, K.T. Yang and R.L. McClain (2001)

[3]. Fractional-order PI Controller Tuning for a System with Delay with Specified Gain and Phase Margins, Mukesh D. Patil, Shanti Sankara, Vishwesh A. Vyawahare (2012).

[4]. Fractional Order PID Controller for a Shell and Tube Heat Exchanger, B. Girirajan and D.Rathikarani (2017)

[5]. James B. Rawlings Yiyang Jenny Wang. A new robust model predictive control method. II: examples. Elsevier,Journal of Process Control 14, page 249 262, 2004.