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Existence Results for Nonlinear Fractional Differential Equation with Nonlocal Strip and Multi-point Boundary Conditions

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Abstract: Objective of this work was to investigate the existence and uniqueness results for nonlinear liouville-caputo fractional differential equation with nonlocal strip and multi-point boundary conditions by applying Banach Fixed point theorem and Krasnoselskii's Fixed point theorem. Main results has been validated by suitable example.

Keywords: Caputo derivative, Fixed point, Fractional differential equation, Multi-point, Nonlocal Integral conditions.

I. INTRODUCTION

Differential equation of fractional order boundary value problems has been studied by many researchers in the past few decades and several results are covered from theoretical development to applications of the subject which can be found in the publications, for instance, see[1-3] and the references therein. The theory of derivatives and integrals of any arbitrary real or complex order has importance in diverse areas of mathematical physics and engineering sciences. The advantage of fractional calculus become apparent in modelling mechanical and electrical properties of real materials as well as in the description of properties of gases, liquids and in rocks. (see[5,6]). One of the reasons for popularity of the subject is the hereditary property of fractional order operators that has let to several developments such as sliding more controls of fractional order chaotic systems[7,8]. We emphasize that the problem involves the study of fractional order system which includes nonlocal and integral boundary conditions, where the systems are useful in describing physical and chemical process inside various domain which are not possible in initial and boundary value problems.

S.K.Ntouyas et.al, studied the existence of solutions for fractional differential inclusions with sum and integral boundary conditions. T.Jankowski have investigated elaborately about boundary problems for fractional differential equations. Recently, A.Cabada et.al, proposednonlinear fractional differential equations with integral boundary value conditions. Also, Y.Sun et.al, studied positive solutions for a class of fractional differential equations with integral boundary conditions. Moreover, J.Deng et.al, analyzed the existence of solutions of initial value problems for nonlinear fractional differential equations. Finally, B.Ahmad et.al, imposedthe existence of solutions for nonlinear fractional q-difference integral equations with two fractional orders and nonlocal four-point boundary conditions

Motivated from the above work, this paper is concerned with the existence and uniqueness results for the boundary value problem with Lioville-Caputo fractional differential equation using boundary conditions given by

$$^{c}D^{\delta}v(x) = h\big(x, v(x)\big), \qquad x \in [0,1], \quad 2 < \delta \leq 3, \tag{1}$$

$$v(0)|_{x=0} = \varphi(v), \quad v^{'}(0)|_{x=0} = \zeta \int_{\lambda_1}^{\mu_1} v^{'}(\tau)\, d\tau,$$

$$v(1)|_{x=0} = \nu \int_{\lambda_2}^{\mu_2} v(\tau)\, d\tau \tag{2}$$

where D^{δ} is the Caputo fractional derivative, $h : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ and $\varphi : C([0,1], \mathbb{R}) \to \mathbb{R}$ are continuous functions, provided$0 < \lambda_1 < \mu_1 < \lambda_2 < \mu_2 < 1,\ \nu, \zeta$ are positive real constants.

The above problem states that the first condition of the value of unknown function at the left terminal(x=0) becomes nonlocal function. The second condition connects the value of the first derivative of the unknown function at the left terminal(x=0) and the first strip occupying the interval (λ_1, μ_1), whereas the third condition connects the value of the unknown function at the right terminal, x=1 and the second strip occupying the interval (λ_2, μ_2).

The organisation of this paper has some definitions from fractional calculus with auxiliary lemmas. Existence results for Lioville-Caputo fractional differential equations with nonlocal integral boundary conditions

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are obtained by Banach and Krasnoselskii's Fixed point theorem. The paper concludes with illustrative and valid example.

II. PRELIMINARIES

Definition 2.1 For a continuous function $h:(a,\infty)\to\mathbb{R}$, the Riemann-Lioville fractional integral of order α is defined as

$$^{RL}I^{\alpha}h(x)=\frac{1}{\Gamma(\alpha)}\int_a^x(x-\sigma)^{\alpha-1}h(\sigma)d\sigma,\qquad x>0,\quad \alpha>0,$$

provided the integral is pointwise on (a,∞).

Definition 2.2 The Riemann-Lioville fractional derivative of order α for a continuous function $h(x)$is defined by

$$^{RL}D^{\alpha}h(x)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n\int_a^x(x-\sigma)^{n-\alpha-1}h(\sigma)d\sigma,\qquad \alpha>0,\quad n-1<\alpha<n$$

where $\Gamma$is the Gamma function, $n=[\alpha]+1$, and $[\alpha]$denotes the integer part of number α, provided the integral is pointwise on (a,∞).

Definition 2.3[4] The Caputo fractional derivative of order α for a continuous function $h:[0,\infty)\to\mathbb{R}$ can be written as

$$^{c}D^{\alpha}h(x)=D^{\alpha}\left(h(x)-\sum_{j=0}^{n-1}\frac{x^j}{j!}h^{(j)}(0)\right),\qquad x>0\quad n-1<\alpha<n.$$

Remark 2.4 If $h(x)\in AC^n([0,\infty))$, then

$$^{c}D^{\alpha}h(x)=\frac{1}{\Gamma(n-\alpha)}\int_0^x(x-\sigma)^{n-\alpha-1}h^n(\sigma)d\sigma=I^{n-\alpha}h^n(x),\qquad \alpha>0,\quad n-1<\alpha<n.$$

Lemma 2.5 For a given $h\in AC[0,1], v\in AC^n([0,1],\mathbb{R})$, the solution of the boundary value problem

$$^{c}D^{\delta}v(x)=h(x),\qquad 2<\delta\le 3,\tag{3}$$

$$v(0)|_{x=0}=\varphi(v),\quad v'(0)|_{x=0}=\zeta\int_{\lambda_1}^{\mu_1}v'(\tau)\,d\tau,\quad v(1)|_{x=0}=\nu\int_{\lambda_2}^{\mu_2}v(\tau)\,d\tau$$

$$v(x)=\int_0^x\frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau)d\tau+\varphi(v)\phi_3(x)+\phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^{\tau}\frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}h(\theta)d\theta\right)d\tau\right]$$
$$+\phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^{\tau}\frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}h(\theta)d\theta\right)d\tau-\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau)d\tau\right]\tag{4}$$

where

$$\vartheta_1=1-\zeta(\mu_1-\lambda_1),\quad \vartheta_2=1-\frac{\nu(\mu_2^2-\lambda_2^2)}{2},$$
$$\vartheta_3=\zeta(\mu_1^2-\lambda_1^2),\quad \vartheta_4=1-\frac{\nu(\mu_2^3-\lambda_2^3)}{3}\tag{5}$$

$$\phi_1(x)=\frac{(\vartheta_4x-\vartheta_2x^2)}{\vartheta},\quad \phi_2(x)=\frac{(\vartheta_3x+\vartheta_1x^2)}{\vartheta},$$
$$\phi_3(x)=1-\phi_2(x)[1-\nu(\mu_2-\lambda_2)]\tag{6}$$

and

$$\vartheta=\vartheta_1\vartheta_4+\vartheta_2\vartheta_3\neq 0\tag{7}$$

Proof : It is well known that the general solution of fractional differential equation (3) can be written as

$$v(x)=\int_0^x\frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau)d\tau+d_0+d_1x+d_2x^2\tag{8}$$

where $d_k\in\mathbb{R}, k=0,1,2,$ are arbitrary constants. Using the boundary conditions in (2), we find that $d_0=\varphi(v)$ and

$$\vartheta_1d_1-\vartheta_3d_2=\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^{\tau}\frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}h(\theta)d\theta\right)d\tau\tag{9}$$

$$\vartheta_2d_1+\vartheta_4d_2=\int_{\lambda_2}^{\mu_2}\left(\int_0^{\tau}\frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}h(\theta)d\theta\right)d\tau-\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau)d\tau-\big(1-\nu(\mu_2-\lambda_2)\big)\varphi(v)\tag{10}$$

where $\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4$are given by (5), solving the systems (9) and (10) for d_1,d_2 and using (7) we get, the solution of (3) is obtained by substituting the value of d_1 and d_2 in (8), we get (3). This completes the proof.

Let $\mathfrak{Q} = \{v : v \in C([0,1], \mathbb{R})\}$ denote the Banach space of all continuous functions from $[0,1]$ into $\mathbb{R}$ endowed with the usual norm defined by $\|v\| = \sup_{x\in[0,1]} |v(x)|$.In view of lemma 2.5, we define an operator $\mathfrak{F} : \mathfrak{Q} \to \mathfrak{Q}$ as

$$(\mathfrak{G}v)(x) = \int_0^x \frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)} h\big(\tau, v(\tau)\big)d\tau + \varphi(v)\phi_3(x) + \phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)} h\big(\theta, v(\theta)\big)d\theta\right)d\tau\right] + \phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)} h\big(\theta, v(\theta)\big)d\theta\right)d\tau - \int_0^1 \frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)} h\big(\tau, v(\tau)\big)d\tau\right] \quad (11)$$

We see that the system (3) has solutions only if the operator equation $v = \mathfrak{G}v$ has fixed points.

III. EXISTENCE RESULTS

In the sequel, we assume that

(G_1) $h : [0,1] \times \mathbb{R} \to \mathbb{R}$ is continuous function and there exist constant $K > 0$ such that

$$|h(x,v) - h(x,u)| \le K\|v-u\|, \quad \forall\, x \in [0,1], \quad v,u \in \mathbb{R}.$$

(G_2) $\varphi : C([0,1], \mathbb{R}) \to \mathbb{R}$ is continuous function with $\varphi(0) = 0$ and there exist constant $k > 0$ such that

$$|\varphi(v) - \varphi(u)| \le k\,\|v-u\|, \quad \forall\, x \in [0,1], \quad v,u \in C([0,1], \mathbb{R}).$$

(G_3) $|h(x,v)| \le \rho(x), \quad \forall\, (x,v) \in [0,1]\times\mathbb{R}$ and $\rho \in C([0,1],\mathbb{R})$with $\|\rho\| = \max_{x\in[0,1]} |\rho(x)|$

For computational convenience, we set

$$\Delta^* = K\varsigma + k\,\hat{\phi}_3\,, \qquad \Omega^* = P\varsigma \quad (12)$$

where

$$\varsigma = \frac{\big(1+\hat{\phi}_2\big) + \hat{\phi}_1\zeta(\mu_1-\lambda_1)^{\delta}}{\Gamma(\delta+1)} + \frac{\hat{\phi}_2\nu(\mu_2-\lambda_2)^{\delta+1}}{\Gamma(\delta+2)}.$$

Theorem 3.1 Assume that the condition $(G_1) - (G_2)$ hold. In addition, it is assumed that $\Delta^* < 1$, where Δ^* is defined by (12). Then there exists atmost one solution for problem (3) on $[0,1]$.

Proof: Define $\sup_{x\in[0,1]} |h(x,0)| = P < \infty$. Selecting $\hat{r} \ge \frac{\Omega^*}{1-\Delta^*}$, we show that $\mathfrak{F}B_{\hat{r}} \subset B_{\hat{r}}$, where

$$\begin{aligned}
\|(\mathfrak{F}v)(x)\| = \sup_{x\in[0,1]}\Bigg|&\int_0^x \frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)} h\big(\tau, v(\tau)\big)d\tau + \varphi(v)\phi_3(x) \\
&+ \phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)} h\big(\theta, v(\theta)\big)d\theta\right)d\tau\right] \\
&+ \phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)} h\big(\theta, v(\theta)\big)d\theta\right)d\tau - \int_0^1 \frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)} h\big(\tau, v(\tau)\big)d\tau\right]\Bigg| \\
\le \sup_{x\in[0,1]}\Bigg\{&\int_0^x \frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)} \big|h\big(\tau, v(\tau)\big) - h(\tau,0) + h(\tau,0)\big|d\tau + |\varphi(v)||\phi_3(x)| \\
&+ \phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)} \big|h\big(\theta, v(\theta)\big) - h(\theta,0) + h(\theta,0)\big|d\theta\right)d\tau\right] \\
&+ \phi_1(x)\Bigg[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)} \big|h\big(\theta, v(\theta)\big) - h(\theta,0) + h(\theta,0)\big|d\theta\right)d\tau \\
&+ \int_0^1 \frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)} \big|h\big(\tau, v(\tau)\big) - h(\tau,0) + h(\tau,0)\big|d\tau\Bigg]\Bigg\} \\
\le (K\|v\| + P)\sup_{x\in[0,1]}&\left\{\int_0^x \frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)} d\tau + \phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)} d\theta\right)d\tau\right]\right. \\
&\left.+ \phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)} d\theta\right)d\tau + \int_0^1 \frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)} d\tau\right]\right\} + k\|v\|\sup_{x\in[0,1]}|\phi_3(x)| \\
\le (K\hat{r} + P)&\left[\frac{\big(1+\hat{\phi}_2\big) + \hat{\phi}_1\zeta(\mu_1-\lambda_1)^{\delta}}{\Gamma(\delta+1)} + \frac{\hat{\phi}_2\nu(\mu_2-\lambda_2)^{\delta+1}}{\Gamma(\delta+2)}\right] + k\hat{r}\hat{\phi}_3 \\
&\le (K\hat{r} + P)\varsigma + k\hat{r}\hat{\phi}_3
\end{aligned}$$

which means that $\mathfrak{F}B_{\hat{r}} \subset B_{\hat{r}}$.

Now, for $v, u \in B_{\hat{r}}$, we obtain

$$\begin{aligned}\|\mathfrak{F}v-\mathfrak{F}u\| \le \sup_{x\in[0,1]}\Bigg\{&\int_0^x \frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)}|h(\tau,v(\tau))-h(\tau,u(\tau))|d\tau+|\varphi(v)-\varphi(u)||\phi_3(x)|\\ &+\phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}|h(\theta,v(\theta))-h(\theta,u(\theta))|d\theta\right)d\tau\right]\\ &+\phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}|h(\theta,v(\theta))-h(\theta,u(\theta))|d\theta\right)d\tau\right.\\ &\left.+\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}|h(\tau,v(\tau))-h(\tau,u(\tau))|d\tau\right]\Bigg\}\\ \le K\|v-u\|\sup_{x\in[0,1]}\Bigg\{&\int_0^x(x-\tau)^{\delta-1}d\tau+\phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}d\theta\right)d\tau\right]\\ +\phi_1(x)\Bigg[\nu&\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}d\theta\right)d\tau+\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}d\tau\Bigg]\Bigg\}+k\|v-u\|\sup_{x\in[0,1]}|\phi_3(x)|\\ \le \Delta^*\|v-u\|.&\end{aligned}$$

Since $\Delta^* \in (0,1)$ by the given assumption, therefore $\mathfrak{F}$ is a contraction. Hence it follows by Banach fixed point theorem that the equation (1) has a unique solution.
The second existence result is based on Krasnoselskii's fixed point theorem.

Lemma 3.2 Let $\mathcal{M}$ be a closed convex and nonempty subset of a Banach space $\mathcal{S}$. Let $\mathfrak{F}_1, \mathfrak{F}_2$ be the operators such that

(i) $\mathfrak{F}_1 v + \mathfrak{F}_2 u \in \mathcal{M}$ whenever $v, u \in \mathcal{M}$;
(ii) $\mathfrak{F}_1$is compact and continuous;
(iii) $\mathfrak{F}_2$is a contraction mapping. Then there exists $w \in \mathcal{M}$ such that $w = \mathfrak{F}_1 w + \mathfrak{F}_2 w$.

Theorem 3.3 Assume that $h : [0,1] \times \mathbb{R} \to \mathbb{R}$and $\varphi : C([0,1],\mathbb{R}) \to \mathbb{R}$ are continuous functions. In addition, assume that the condition $(G_1) - (G_3)$ hold.then the boundary value problem (1) has at least one solution on $[0,1]$ if

$$K\left(\frac{\hat{\phi}_2+\hat{\phi}_1\zeta(\mu_1-\lambda_1)^\delta}{\Gamma(\delta+1)}+\frac{\hat{\phi}_2\nu(\mu_2-\lambda_2)^{\delta+1}}{\Gamma(\delta+2)}\right)+k\hat{\phi}_3<1 \qquad (13)$$

Proof: we fix

$$\hat{r} \ge \frac{\varsigma\|\rho\|}{1-\widehat{\phi_3}}. \qquad (14)$$

Consider $B_{\hat{r}} = \{v \in \mathfrak{Q} : \|v\| \le \hat{r}\}$. Define the operators $\mathfrak{F}_1$and $\mathfrak{F}_2$ on $B_{\hat{r}}$ as

$$(\mathfrak{F}_1 v)(x) = \int_0^x \frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau,v(\tau))d\tau$$

$$\begin{aligned}(\mathfrak{F}_2 v)(x) = \varphi(v)\phi_3(x)&+\phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}h(\theta,v(\theta))d\theta\right)d\tau\right]\\ &+\phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}h(\theta,v(\theta))d\theta\right)d\tau-\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau,v(\tau))d\tau\right]\end{aligned}$$

For $v, u \in B_{\hat{r}}$, it follows from (14) that

$$\begin{aligned}\|\mathfrak{F}v\| &\le \left[\frac{(1+\hat{\phi}_2)+\hat{\phi}_1\zeta(\mu_1-\lambda_1)^\delta}{\Gamma(\delta+1)}+\frac{\hat{\phi}_2\nu(\mu_2-\lambda_2)^{\delta+1}}{\Gamma(\delta+2)}\right]\|\rho\|+\|v\|\hat{\phi}_3\\ &\le \varsigma\|\rho\|+\hat{r}\hat{\phi}_3\le\hat{r}\end{aligned}$$

Thus,

$$\begin{aligned}\|\mathfrak{F}_2 v-\mathfrak{F}_2 u\| \le \sup_{x\in[0,1]}\Bigg\{&|\varphi(v)-\varphi(u)||\phi_3(x)|\\ &+\phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}|h(\theta,v(\theta))-h(\theta,u(\theta))|d\theta\right)d\tau\right]\\ &+\phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau\frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}|h(\theta,v(\theta))-h(\theta,u(\theta))|d\theta\right)d\tau\right.\\ &\left.+\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}|h(\tau,v(\tau))-h(\tau,u(\tau))|d\tau\right]\Bigg\}\end{aligned}$$

$$\leq K\|v-u\| \sup_{x\in[0,1]}\left\{\phi_1(x)\left[\zeta\int_{\lambda_1}^{\mu_1}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-2}}{\Gamma(\delta-1)}d\theta\right)d\tau\right]\right.$$
$$\left.+\phi_1(x)\left[\nu\int_{\lambda_2}^{\mu_2}\left(\int_0^\tau \frac{(\tau-\theta)^{\delta-1}}{\Gamma(\delta)}d\theta\right)d\tau+\int_0^1\frac{(1-\tau)^{\delta-1}}{\Gamma(\delta)}d\tau\right]\right\}+k\|v-u\|\sup_{x\in[0,1]}|\phi_3(x)|$$
$$\leq\left[K\left(\frac{\hat{\phi}_2+\hat{\phi}_1\zeta(\mu_1-\lambda_1)^\delta}{\Gamma(\delta+1)}+\frac{\hat{\phi}_2\nu(\mu_2-\lambda_2)^{\delta+1}}{\Gamma(\delta+2)}\right)+k\hat{\phi}_3\right]\|v-u\|$$

Thus, it follows that $\mathfrak{F}_2$ is a contraction mapping in $B_{\hat{r}}$. The continuity of h implies that the operator $\mathfrak{F}_1$is continuous. Also, $\mathfrak{F}_1$is uniformly bounded on $B_{\hat{r}}$ as

$$\|\mathfrak{F}v\|\leq \sup_{x\in[0,1]}\left\{\int_0^x\frac{(x-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau,v(\tau))d\tau\right\}\leq\frac{\|\rho\|}{\Gamma(\delta+1)}.$$

Next, we prove the compactness of the operator $\mathfrak{F}_1$. Let $x_1,x_2\in[0,1]$ and setting $\sup_{x\in[0,1]}|h(x,v)|=\hat{h}$. We have

$$|\mathfrak{F}_1(v)(x_2)-\mathfrak{F}_1(v)(x_1)|\leq\left|\int_0^{x_1}\frac{(x_2-\tau)^{\delta-1}-(x_1-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau,v(\tau))d\tau+\int_{x_1}^{x_2}\frac{(x_2-\tau)^{\delta-1}}{\Gamma(\delta)}h(\tau,v(\tau))d\tau\right|$$
$$\leq\frac{\hat{h}}{\Gamma(\delta+1)}\left[|(x_2-x_1)^\delta|+|x_2^\delta-x_1^\delta|\right]$$

which is independent of v and tends to zero as $x_2\to x_1$. Thus, $\mathfrak{F}_1$ is relatively compact on $B_{\hat{r}}$. Hence, by the Arzela-Ascoli theorem, $\mathfrak{F}_1$is compact on $B_{\hat{r}}$. Thus all assumptions of Lemma 3.2 implies that the boundary value problem (1) has at least one solution on $[0,1]$. This completes the proof.

IV. EXAMPLES

Example 4.1 Consider the following BVP of FDEs given by

$$^cD^{\frac{7}{3}}v(x)=\frac{x}{4}+\frac{1}{7(x+1)^2}\left(\frac{|v(x)|}{1+|v(x)|}\right),\qquad x\in[0,1],\quad 2<\delta\leq 3, \tag{15}$$

$$v(0)=\frac{1}{4}\tan^{-1}\left(v\left(\frac{1}{5}\right)\right),\quad v'(0)=\int_{\frac{1}{9}}^{\frac{1}{7}}v'(\tau)\,d\tau,\quad v(1)=\int_{\frac{1}{3}}^{\frac{1}{2}}v(\tau)\,d\tau \tag{16}$$

Here, $\delta=\frac{7}{3},\zeta=1,\nu=1,\lambda_1=\frac{1}{9},\mu_1=\frac{1}{7},\lambda_2=\frac{1}{3},\mu_2=\frac{1}{2},h(t,v)=\frac{x}{4}+\frac{1}{3(x+1)^2}\left(\frac{|v(x)|}{1+|v(x)|}\right)$, $\varphi(v)=\frac{1}{4}\tan^{-1}\left(v\left(\frac{1}{5}\right)\right)$. Clearly,we obtain the inequalities $|h(x,v)-h(x,u)|\leq\frac{1}{6}\|v-u\|$, $|\varphi(v)-\varphi(u)|\leq\frac{1}{4}\|v-u\|$, therefore, $(G_1)$and (G_2) are satisfied with $K=\frac{1}{14}$ and $=\frac{1}{10}$. Besides we deduce $\varsigma\approx 0.71164,\phi_1\approx 0.020796,\phi_2\approx 0.97633,\phi_3\approx 0.18639$ and $\Delta^*\approx 0.16519<1.$ Thus, All conditions of theorem 3.1 are satisfied. Therefore, by theorem 3.1 we conclude that system (15) has a unique solution on $x\in[0,1]$.

V. CONCLUSION

By the above results it is evident that the existence and uniqueness obeys for a new boundary value problem ofnonlinear fractional differential equations supplemented with nonlocal strip integral boundary conditions. The results of this paper opens the gateway for the reader with abundant ideas with certain appropriate values of the parameters involved in the problem.

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